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Longitudinal surface curvature effect in magnetohydrodynamics

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SUMMARY

The two-dimensional motion of an incompressible and electrically conducting fluid past an electrically insulated body surface (having curvature) is studied for a given $O(1)$ basic flow and magnetic field, when (i) the applied magnetic field is aligned with the velocity in the basic flow, and (ii) the applied magnetic field is within the body surface. ($O(1)$ and $O(Re^{-\frac{1}{2}})$ mean the first and second order approximations respectively in an expansion scheme in powers of $Re^{-\frac{1}{2}}$, Re being the Reynolds number.) The technique of matched asymptotic expansions is used to solve the problem. The governing partial differential equations to $O(Re^{-\frac{1}{2}})$ boundary layer approximation are found to give similarity solutions for a family of surface curvature and pressure gradient distributions in case (i), and for uniform basic flow with analytic surface curvature distributions in case (ii). The equations are solved numerically.

In case (i) it is seen that the effect of the magnetic field on the skin-friction-correction due to the curvature is very small. Also the magnetic field at the wall is reduced by the curvature on the convex side.

In case (ii) the magnetic field significantly increases the skin-friction-correction due to the curvature. The effect of the magnetic field on the $O(1)$ and $O(Re^{-\frac{1}{2}})$ skin-friction coefficients increases with the increase of the electrical conductivity of the fluid. Also, at higher values of the magnetic pressure, moderate changes in the electrical conductivity do not influence the correction to the skin-friction significantly.

1. Introduction

The theoretical study of magnetohydrodynamic flow around obstacles and at system boundaries is important mainly because the ambient magnetic field significantly influences the structure of the boundary layer.

In hydrodynamics, Van Dyke [1] formulated the problem of higher order approximations in boundary layer theory. He distinguished five additive effects to the $O(Re^{-\frac{1}{2}})$ approximation, where Re is the Reynolds number. Each of these effects has been separately studied. Narasimha and Ozha [2] have studied the longitudinal surface curvature effect. They have given a thorough review of the work on the problem. Honda and Kiyokawa [3] have studied the second order boundary layer problem in a different way.

To the best of the knowledge of the authors, no such general formulation to obtain higher order approximations to MHD boundary layers exists. In this work an attempt is made to investigate the second order approximation to MHD boundary layers by considering the effect of the longitudinal surface curvature. Two magnetic field configurations are considered, *viz.* (i) the magnetic field aligned with the velocity field in the basic flow and (ii) the magnetic field within the body surface (i.e. flow over a magnetised surface).

Gribben [4] has investigated the Prandtl-type MHD boundary layer flow with non-zero pressure gradient and the magnetic field aligned to the velocity field in the basic flow. Zigulev [5] and Glauert [6] have studied such a boundary layer on a magnetised surface for a uniform basic flow.

In this work, we use the matched asymptotic expansions technique to solve the problem. Two sets of complementary expansions in powers of $Re^{-\frac{1}{2}}$ to be valid in the regions far away from the body surface (outer) and very close to it (inner) are constructed for the field variables.

These expansions are suitably matched in the region of their common validity to give the additional boundary conditions for the boundary layer equations. The boundary layer partial differential equations are transformed into ordinary differential equations by a similarity analysis. This is possible for a family of pressure gradients and surface curvature distributions in the case (i), and for a uniform basic flow and analytic surface curvature distribution in the case (ii). The equations are solved numerically on an IBM 7044 digital computer at I.I.T. Kanpur (India) by the Runge–Kutta–Gill method for moderate values of the parameters. The results are presented in the form of tables and graphs at the end of the paper.

2. Formulation of the problem with aligned magnetic field (Case (1))

Let us consider the axially directed steady flow of an electrically conducting, incompressible and viscous fluid past a smooth electrically insulated solid surface of revolution in the presence of a magnetic field. Let $(\bar{X}, \bar{Y}, \bar{Z})$ be rectangular Cartesian coordinates of any point on the body-surface and let the body-axis lie along the axis of $\bar{X}$. In the limit when the $\bar{Z}$-coordinate of every point tends to infinity, we obtain the two-dimensional case, which is studied here. The governing equations for the problem are the equations of continuity for the velocity and magnetic fields and the momentum conservation and the magnetic diffusion equations. Expressed in non-dimensional vector form [7] they are,

$$\operatorname{div} \boldsymbol{Q} = 0\,, \tag{1}$$

$$\boldsymbol{Q} \cdot \operatorname{grad} \boldsymbol{Q} = -\operatorname{grad} \Pi + S\boldsymbol{H} \cdot \operatorname{grad} \boldsymbol{H} - \varepsilon^2 \operatorname{curl}\operatorname{curl} \boldsymbol{Q}\,, \tag{2}$$

$$\operatorname{div} \boldsymbol{H} = 0 \tag{3}$$

and

$$\boldsymbol{Q} \times \boldsymbol{H} = (\varepsilon^2/Pm) \operatorname{curl} \boldsymbol{H}\,, \tag{4}$$

where

$$S = (\mu^* H_\infty^{*2})/(\rho^* Q_\infty^{*2})\,, \quad Pm = \sigma^* \mu^* \nu^*\,,$$

$$\varepsilon^2 = Re^{-1} = \nu^*/(Q_\infty^* L^*)\,, \quad \Pi = P + SH^2/2\,.$$

$\boldsymbol{Q}$ and $\boldsymbol{H}$ represent the non-dimensional velocity and magnetic field vectors, P the non-dimensional pressure, L^* some characteristic length and σ^*, μ^*, ν^* are the dimensional electrical conductivity, the magnetic susceptibility and the kinematic viscosity. Starred quantities are dimensional and the subscript represents the value in the region specified by it.

The boundary conditions for the problem are

$$Q_{\tan} \to 1\,, \quad Q_{\text{norm}} \to 0\,, \quad H_{\tan} \to 1\,, \quad H_{\text{norm}} \to 0\,, \quad \text{at up stream infinity}\,, \tag{5a}$$

$$\boldsymbol{Q} = 0\,, \quad H_{\text{norm}} = 0\,, \quad \text{at the body surface}\,, \tag{5b}$$

where the subscripts "tan" and "norm" denote the components tangential and normal to the body surface respectively.

As is usually done in the method of matched asymptotic expansions, we construct outer expansions of the field variables, to be valid in the basic flow, in the limit $\varepsilon \to 0$, with other variables fixed. Thus

$$\boldsymbol{Q} = \boldsymbol{Q}_1 + \varepsilon \boldsymbol{Q}_2 + \dots\,, \tag{6}$$

and similarly for P, H and Π, where $\boldsymbol{Q}_1$, $\boldsymbol{Q}_2$, etc. are functions of the coordinates only.

We substitute the outer expansions in the governing equations and collect terms with equal powers of ε. Thus equation (4) gives

$$\boldsymbol{Q}_1 \times \boldsymbol{H}_1 = 0 = \boldsymbol{Q}_2 \times \boldsymbol{H}_2 = \dots\,. \tag{7}$$

From (7) and (5), we obtain

$$\boldsymbol{Q}_1 = \boldsymbol{H}_1\,, \quad \boldsymbol{Q}_2 = \boldsymbol{H}_2\,, \quad \text{etc.} \tag{8}$$

The arbitrary scalar constants multiplying $\boldsymbol{H}_1$, $\boldsymbol{H}_2$, etc. in equations (8) can be taken to be unity without any loss of generality. Thus in the outer flow the magnetic lines of force are completely aligned with the flow field to all orders in ε.

It will be useful at this stage to introduce a system of orthogonal curvilinear coordinates (X, Y) consisting of lines parallel to the body surface and orthogonal trajectories normal to the body surface, with the origin of coordinates at the front stagnation point. We shall refer to this system as the curvature coordinate system. The non-dimensional surface curvature $K(X) = 1/R(X)$ is measured positive when the wall is convex outwards. The line element of length in this system is given by

$$dS^2 = (1+KY)^2 dX^2 + dY^2 . \tag{9}$$

Equations (1) through (4) can now be expressed in component form by using the generalized orthogonal form of the vector operations involved and the relation (9). Given here for the first time, they are

$$(1+KY)^{-1} U_X + V_Y + K(1+KY)^{-1} V = 0 , \tag{10}$$

$$\begin{aligned}(1+KY)^{-1} UU_X + VU_Y + K(1+KY)^{-1} UV = \\ -(1+KY)^{-1} \Pi_X + S\{(1+KY)^{-1} MM_X + NM_Y + K(1+KY)^{-1} MN\} \\ +\varepsilon^2 \Big[(1+KY)^{-2} U_{XX} + U_{YY} + K(1+KY)^{-1} U_Y + 2K(1+KY)^{-2} V_X \\ -K^2(1+KY)^{-2} U - (1+KY)^{-3} Y \frac{dK}{dX} U_X + (1+KY)^{-2} \frac{dK}{dX} V\Big],\end{aligned} \tag{11a}$$

$$\begin{aligned}(1+KY)^{-1} UV_X + VV_Y - K(1+KY)^{-1} U^2 = \\ -\Pi_Y + S\{(1+KY)^{-1} MN_X + NN_Y - K(1+KY)^{-1} M^2\} \\ +\varepsilon^2 \Big[(1+KY)^{-2} V_{XX} + V_{YY} + K(1+KY)^{-1} V_Y - 2K(1+KY)^{-2} U_X \\ -K^2(1+KY)^{-2} V - (1+KY)^{-3} \frac{dK}{dX} U - (1+KY)^{-3} Y \frac{dK}{dX} V_X\Big] ,\end{aligned} \tag{11b}$$

$$(1+KY)^{-1} M_X + N_Y + K(1+KY)^{-1} N = 0 , \tag{12}$$

$$UN - VM = \frac{\varepsilon^2}{Pm} [(1+KY)^{-1} N_X - M_Y - K(1+KY)^{-1} M] , \tag{13}$$

where (U, V) and (M, N) are the components of the velocity and magnetic field respectively and the letter suffixes denote the partial differentiation.

The hydrodynamic equations in the curvature coordinates have been given by Schlichting [8].

The outer expansions (6) can be broken into the component form and then substituted in equations (10) through (13). Collecting equal powers of ε we obtain outer equations for different orders. In general these equations satisfy only the outer boundary conditions (5a). From the first order outer expansions and their boundary conditions we can obtain

$$\begin{aligned}&\lim_{Y\to 0} \Pi_{1Y} = K(U_1^2 - SM_1^2) , \\ &\lim_{Y\to 0} U_{1Y} = -KU_1 \ ; \ \lim_{Y\to 0} M_{1Y} = -KM_1 ,\end{aligned} \tag{14}$$

with $U_1 = M_1$ from (7).

Using lower case letters to denote the variables in the boundary layer, we define an inner limit, $\varepsilon \to 0$, as $x = X$ and $y = Y/\varepsilon$ are fixed and make the following expansions.

$$u(x, y, \varepsilon) \sim u_1(x, y) + \varepsilon u_2(x, y) + O(\varepsilon^2) , \tag{15a}$$

$$v(x, y, \varepsilon) \sim \varepsilon v_1(x, y) + \varepsilon^2 v_2(x, y) + O(\varepsilon^3) , \tag{15b}$$

$$\pi(x, y, \varepsilon) \sim \pi_1(x, y) + \varepsilon \pi_2(x, y) + O(\varepsilon^2) . \tag{15c}$$

The expansions for m and n are similar to those for u and v respectively. Substituting (15) in equations (10) through (13) and collecting equal powers of ε, we obtain

$$u_{1x}+v_{1y}=0\,, \tag{16a}$$

$$u_{1yy}-(u_1u_{1x}+v_1u_{1y})+S(m_1m_{1x}+n_1m_{1y})-\pi_{1x}=0\,, \tag{16b}$$

$$\pi_{1y}=0\,, \tag{16c}$$

$$m_{1x}+n_{1y}=0\,, \tag{16d}$$

$$m_{1y}+Pm(u_1n_1-v_1m_1)=0\,, \tag{16e}$$

$$u_{2x}+(v_2+Kyv_1)_y=0\,, \tag{17a}$$

$$u_{2yy}-(u_1u_{2x}+u_2u_{1x}+v_1u_{2y}+v_2u_{1y})+S(m_1m_{2x}+m_2m_{1x}+n_1m_{2y}+n_2m_{1y})= \\ =\pi_{2x}+K\{y(v_1u_{1y}-u_{1yy}-Sn_1m_{1y})+u_1v_1-u_{1y}-Sm_1n_1\}\,, \tag{17b}$$

$$\pi_{2y}=K(u_1^2-Sm_1^2)\,, \tag{17c}$$

$$m_{2x}+(n_2+Kyn_1)_y=0\,, \tag{17d}$$

$$m_{2y}+Pm(u_2n_1+u_1n_2-v_2m_1-v_1m_2)+ \\ K[y\{m_{1y}+Pm(u_1n_1-v_1m_1)\}+m_1]=0\,. \tag{17e}$$

These equations satisfy the boundary conditions at the body surface, i.e.

$$y=0:\ u_1=v_1=n_1=0\,, \tag{18a}$$

$$y=0:\ u_2=v_2=n_2=0\,, \tag{19a}$$

and in general, they do not satisfy conditions at infinity.

To get complete solutions of the equations (16) and (17), additional boundary conditions are needed. These are obtained by matching the inner and outer expansions in their region of common validity. This is done by taking the inner asymptotic expansion of the outer solution and comparing this to appropriate orders with the inner asymptotic expansion for large y [9]. Thus we have

$$V_1(X,0)=0\,, \tag{18b}$$

$$N_1(X,0)=0\,, \tag{18c}$$

$$u_1(x,\infty)=U_1(X,0)\,, \tag{18d}$$

$$m_1(x,\infty)=M_1(X,0)\,, \tag{18e}$$

$$\pi_1(x,\infty)=\Pi_1(X,0)\,, \tag{18f}$$

$$u_2(x,y)\approx -KyU_1(X,0)+U_2(X,0)\,,\ \text{as}\ y\to\infty\,, \tag{19b}$$

$$m_2(x,y)\approx -KyM_1(X,0)+M_2(X,0)\,,\ \text{as}\ y\to\infty\,, \tag{19c}$$

$$\pi_2(x,y)\approx Ky[U_1^2(X,0)-SM_1^2(X,0)]+\Pi_2(X,0)\,,\ \text{as}\ y\to\infty\,. \tag{19d}$$

In deriving these equations we have assumed that the Taylor series expansions of the outer variables near $Y=0$, exist, and we have also used relations (14).

The terms $U_2(X,0)$, $M_2(X,0)$ and $\Pi_2(X,0)$ in the above boundary conditions arise due to the displacement thickness effect. They are of the same order as the other terms. However, equations (17) are linear and therefore allow superposition. Thus following a suggestion by Rott and Lenard [10], the displacement thickness effect can be removed.

From equations (17c) and (19d), we obtain the hydromagnetic pressure as

$$\pi_2(x,y)=K\int_0^y(u_1^2-Sm_1^2)dy-K\int_0^\infty\{(u_1^2-U_1^2)-S(m_1^2-M_1^2)\}\,dy\,. \tag{20}$$

We combine equations (16b, c) and (20). Thus

$$u_{2yy}-(u_1u_{2x}+u_2u_{1x}+v_1u_{2y}+v_2u_{1y})+S(m_1m_{2x}+m_2m_{1x}+n_1m_{2y}+n_2m_{1y})=$$
$$=\left[K\int_0^y(u_1^2-Sm_1^2)dy-K\int_0^\infty\{(u_1^2-U_1^2)-S(m_1^2-M_1^2)\}dy\right]_x+$$
$$K[y(v_1u_{1y}-u_{1yy}-Sn_1m_{1y})+u_1v_1-u_{1y}-Sm_1n_1]\,. \quad (21)$$

The boundary conditions for the $O(\varepsilon)$ equations are

$$y=0:\quad u_2=v_2=0\,,\ n_2=0\,, \quad (22a)$$
$$y\to\infty:\ u_2\to -KyU_1\,,\ m_2\to -KyM_1\,. \quad (22b)$$

This completes the formulation of $O(Re^{-\frac{1}{2}})$ problem.

3. Similarity analysis for case (1)

The partial differential equations (16) and (17, 21) can be transformed into ordinary differential equations by the similarity method. Thus we introduce the following transformations:

$$\xi=\int_0^x U_1dx\ ;\ \eta=(2\xi)^{-\frac{1}{2}}U_1y\ ;\ u_1=U_1f_1'\,,$$
$$v_1=-(2\xi)^{-\frac{1}{2}}U_1\{f_1+(\beta-1)\eta f_1'\}\ ;\ m_1=M_1g_1'\,,$$
$$n_1=-(2\xi)^{-\frac{1}{2}}M_1\{g_1+(\beta-1)\eta g_1'\}\ ;\ u_2=U_1f_2'\,,$$
$$v_2=-(2\xi)^{-\frac{1}{2}}U_1[f_2+(\beta-1)\eta f_2'-k\eta\{f_1+(\beta-1)\eta f_1'\}]\,,$$
$$m_2=M_1g_2'\,,\ n_2=-(2\xi)^{-\frac{1}{2}}M_1[g_2+(\beta-1)\eta g_2'-k\eta\{g_1+(\beta-1)\eta g_1'\}]\,,$$
$$\beta=2\frac{d\ln U_1}{d\ln\xi}\,,\quad K=(2\xi)^{-\frac{1}{2}}U_1k\,,$$

with $U_1=M_1$, and primes denoting differentiation with respect to η. Furthermore, β and k are assumed to be constants.

The equations (16a, d) and (17a, d) are identically satisfied by these transformations. Substituting the transformations in the equations (16b, c, e) and (17e, 21), we obtain

The $O(1)$ similarity equations:

$$f_1'''+f_1f_1''+\beta(1-f_1'^2)=S\{g_1g_1''+\beta(1-g_1'^2)\}\,, \quad (23a)$$
$$g_1''+Pm(f_1g_1'-f_1'g_1)=0\,. \quad (23b)$$

The $O(\varepsilon)$ similarity equations:

$$f_2'''+f_1f_2''+f_1''f_2-S(g_1g_2''+g_1''g_2)+2\beta(Sg_1'g_2'-f_1'f_2')=$$
$$=k[(\beta-1)/(\beta+1)(f_1''+f_1f_1'-Sg_1g_1')+2\beta/(\beta+1)\cdot\{\beta(1-S)\eta+(1-S)\lambda\}-\eta f_1''']\,, \quad (24a)$$

$$g_2''-Pm(g_1f_2'+g_2f_1'-f_1g_2'-g_1'f_2)+k(\eta g_1''+g_1')=0\,, \quad (24b)$$

where

$$\lambda=\lim_{\eta\to\infty}(\eta-f_1)=\lim_{\eta\to\infty}(\eta-g_1)\,.$$

The $O(\varepsilon)$ pressure gradient has been evaluated by using equation (23a). The boundary conditions (18) and (22) become

$$\eta=0:\ f_1=f_1'=g_1=0\,,$$
$$\eta\to\infty:\ f_1'\to 1\,,\ g_1'\to 1\,, \quad (25)$$

$$\eta = 0: \quad f_2 = f_2' = g_2 = 0\,,$$
$$\eta \to \infty: f_2' \to -k\eta\;;\; g_2' \to -k\eta\,. \tag{26}$$

In (24) and (26), k can be absorbed in f_2 and g_2 so that solutions are obtained for f_2/k and g_2/k:

4. Solution and results for case (1)

Equations (23) and (24) are strongly coupled, and in addition equations (23) are also nonlinear. An orthodox approach like power series expansion to get an analytical solution of these equations is not convenient because the $O(\varepsilon)$ equations give complicated expressions. However, numerical methods of solution can be conveniently used. The equations have been solved simultaneously by the Runge–Kutta method with Gill's improvement on an IBM 7044 high speed digital computer at I.I.T., Kanpur (India) for specific values of the parameters β, Pm and S (< 0.5). The solutions are shown in Figs. 1 and 2 and in Table 1 at the end of the paper. The results are believed to be correct up to six decimals. Even then an error of a few units in the sixth decimal cannot be ruled out.

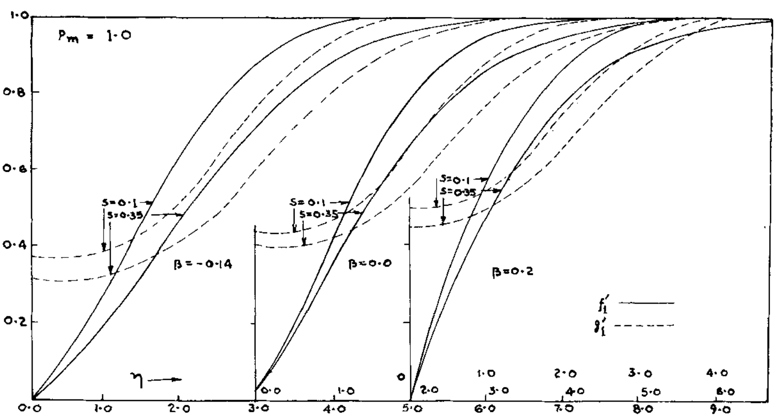

Figure 1. MHD flow with aligned magnetic field. Boundary layer $O(1)$ velocity and magnetic field profiles.

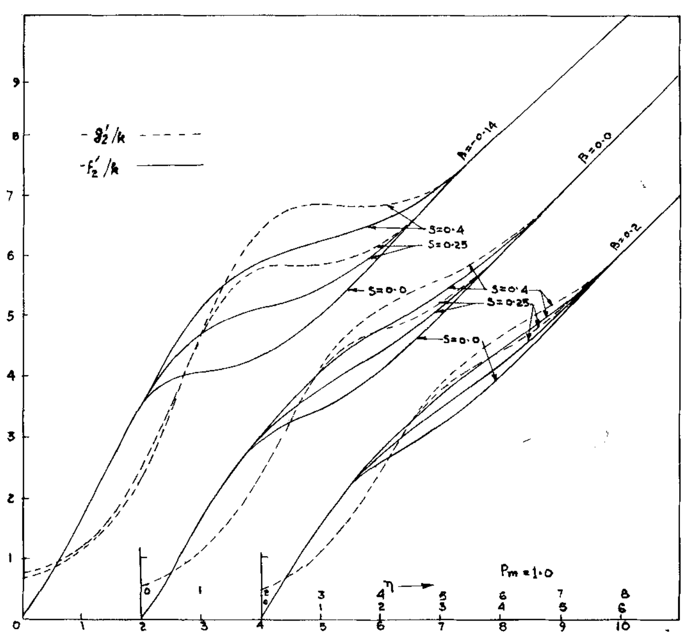

Figure 2. MHD flow with aligned magnetic field. $O(\varepsilon)$ velocity and magnetic field profiles.

TABLE 1

Variation of the skin friction coefficient and magnetic field at the surface (case (i))

β	Pm	S	$f_1''(0)$	$g_1'(0)$	$-f_2''(0)/k$	$-g_2'(0)/k$
−0.14	0.5	0.00	0.239736	0.459440	1.178165	0.562200
		0.10	0.214402	0.439399	1.176294	0.613046
		0.20	0.183536	0.416083	1.170551	0.676277
		0.25	0.170403	0.402848	1.169836	0.717144
	1.0	0.10	0.215640	0.366959	1.178021	0.592748
		0.20	0.189795	0.346843	1.177478	0.645190
		0.30	0.161719	0.323644	1.176534	0.712870
	5.0	0.10	0.221111	0.226500	1.174196	0.474990
		0.20	0.201799	0.214926	1.174482	0.499863
		0.25	0.191859	0.208759	1.174544	0.513889
0.00	0.5	0.00	0.469600	0.531683	1.446967	0.429849
		0.10	0.441773	0.516076	1.445577	0.458324
		0.20	0.411853	0.498452	1.443442	0.491895
		0.30	0.379446	0.478289	1.440103	0.532123
		0.40	0.344025	0.454836	1.436156	0.581970
	1.0	0.10	0.440782	0.442467	1.444451	0.458881
		0.20	0.410076	0.426030	1.440893	0.488426
		0.30	0.377163	0.407496	1.436866	0.523247
		0.50	0.302809	0.361591	1.419676	0.616991
	5.0	0.10	0.441723	0.290364	1.443433	0.397875
		0.20	0.412401	0.278851	1.438974	0.415549
		0.30	0.381390	0.266202	1.432900	0.435724
		0.40	0.348347	0.252135	1.429186	0.460178
0.20	1.0	0.00	0.668927	0.504226	1.645558	0.376213
		0.10	0.650547	0.497144	1.644814	0.386185
		0.20	0.611786	0.481714	1.643914	0.410342
		0.30	0.569932	0.464246	1.642127	0.438836
		0.40	0.524317	0.441170	1.639175	0.474657
		0.50	0.474006	0.420640	1.634189	0.516667
	5.0	0.05	0.668027	0.344745	1.644244	0.345630
		0.15	0.629094	0.333329	1.641732	0.360544
		0.20	0.608756	0.327204	1.639330	0.370190
		0.30	0.560071	0.313968	1.634946	0.389282
		0.40	0.520238	0.299130	1.629327	0.411217

The solutions of the Falkner–Skan equations, given by Smith [11], were used to provide the guess for the initial value $f_1''(0)$. Sensible guesses for the other missing initial values and the small increment in S (which was reduced with the increase in the ground value of S) with other parameters fixed, give rapid convergence.

In Figs. 1 and 2, the boundary layer velocity and magnetic field profiles for different values of β and S and for $Pm=1$ are shown. The variations of the skin friction and the magnetic field at the body surface with S, for different values of β and Pm, are given in Table 1.

To the best of the knowledge of the authors, solutions to even the $O(1)$ boundary layer problem (i.e. equations (23)) are not available for moderate values of Pm and non-zero values of β. In [4], solutions for very large and very small values of Pm have been obtained.

As expected, the magnetic field reduced the $O(1)$ skin friction. The $O(1)$ magnetic field at the wall also reduces with the increase in the value of S. This is similar to the result derived in [4] for large values of Pm.

The magnetic field has a weak influence on the $O(\varepsilon)$ correction to the skin friction due to the surface curvature. It opposes the surface curvature by reducing the $O(\varepsilon)$ correction to the skin friction.

As pointed out in [2], the dominant reason for the reduction of skin friction on the convex side appears to be that in the irrotational outer flow, the velocity tends to decrease away from the surface. Also from equation (20) it can be seen that the hydrodynamic and magnetic parts of the $O(\varepsilon)$ hydrodynamic pressure are opposed to each other. One, therefore, expects the magnetic field to reduce the $O(\varepsilon)$ skin-friction-correction.

A moderate change in the value of Pm does not affect significantly the $O(1)$ and $O(\varepsilon)$ skin friction coefficients. The $O(1)$ and $O(\varepsilon)$ magnetic field values at the wall decrease with an increase in Pm.

5. Flow past a magnetised surface (case (11))

Zigulev [5] has formulated the boundary layer equations for the MHD flow past a surface, magnetised by a passing current through a closely wound solenoid around it. We consider the effect of body surface curvature on such a boundary layer flow. Equations (1) through (4) are the basic equations for the problem, except that in this case S will have a different meaning since the magnetic field is prescribed at the body surface and not in the free stream. The boundary conditions for this case are

$$\boldsymbol{Q} = 0\,,\ \boldsymbol{H}_{\tan} = 1 \quad \text{at the body surface}\,, \tag{27a}$$

$$\boldsymbol{Q}_{\tan} \to 1\ ;\ \boldsymbol{Q}_{\text{norm}} \to 0\ ;\ \boldsymbol{H} \to 0 \ \text{at upstream infinity}\,. \tag{27b}$$

Making outer expansions of the field variables, similar to (6), and substituting them in (4), we obtain

$$\boldsymbol{Q}_1 \times \boldsymbol{H}_1 = 0 = \boldsymbol{Q}_2 \times \boldsymbol{H}_2 = \ldots\,, \tag{28}$$

From (27b) and (28), we have

$$\boldsymbol{H}_1 = 0 = \boldsymbol{H}_2 = \ldots\,. \tag{29}$$

Following exactly the procedure outlined in case (i), we can obtain the partial differential equations for the boundary layer to $O(\varepsilon)$. They are the same as equations (16) and (17, 21), except that in this case $M_1 = 0$. The boundary conditions to be satisfied by these equations are

$$\begin{aligned} &y = 0: \quad u_1 = v_1 = 0\,,\ m_1 = 1\,,\ u_2 = v_2 = m_2 = 0\,,\\ &y \to \infty\ :\ u_1 \to U_1\,,\ m_1 \to 0\,,\ u_2 \to -KyU_1\,,\ m_2 \to 0\,. \end{aligned} \tag{30}$$

The similarity analysis for the problem is possible only for a uniform outer flow. Thus we set π_{1x} equal to zero in equation (16b), and introduce the following similarity transformations:

$$\begin{aligned} &\eta = (2x)^{-\frac{1}{2}} U_1 y\,,\ K = k(2x)^{-\frac{1}{2}} U_1\,,\\ &u_1 = U_1 f_1'\,,\ v_1 = -(2x)^{-\frac{1}{2}} U_1 (f_1 - \eta f_1')\\ &m_1 = g_1'\,,\ n_1 = -(2x)^{-\frac{1}{2}} (g_1 - \eta g_1')\,,\\ &u_2 = U_1 f_2'\,,\ v_2 = -(2x)^{-\frac{1}{2}} U_1 \{f_2 - \eta f_2' - k\eta (f_1 - \eta f_1')\}\,,\\ &m_2 = g_2'\,,\ n_2 = -(2x)^{-\frac{1}{2}} \{g_2 - \eta g_2' - k\eta (g_1 - \eta g_1')\}\,, \end{aligned}$$

where f_1, g_1, f_2, g_2 are the functions of η only, primes denote differentiation with respect to η, and k is a constant.

We substitute these transformations in the boundary layer equations. The resulting equations are (23) and (24) with $\beta = 0$. Here we define $S = (\mu^* H_0^{*2})/(\rho^* Q_\infty^{*2} U_1^2)$, or otherwise U_1 can be conveniently taken to be unity. The boundary conditions (30) become

$$\begin{aligned} &\eta = 0: \quad f_1' = f_1 = 0\ ;\ g_1' = 1\ ;\ f_2 = f_2' = g_2' = 0\,,\\ &\eta \to \infty\ :\ f_1' \to 1\ ;\ g_1' \to 0\ ;\ f_2' \to -k\eta\ ;\ g_2' \to 0\,. \end{aligned} \tag{31}$$

Equations (23) and (24) with $\beta = 0$ and boundary conditions (31) are solved numerically on a computer as in case (i). The solutions are given in Figs. 3 and 4 and in Table 2.

To the best of the knowledge of the authors, the solution of even the $O(1)$ boundary layer problem for moderate values of Pm is not available.

Figs. 3 and 4 show the $O(1)$ and $O(\varepsilon)$ boundary layer velocity and magnetic field profiles

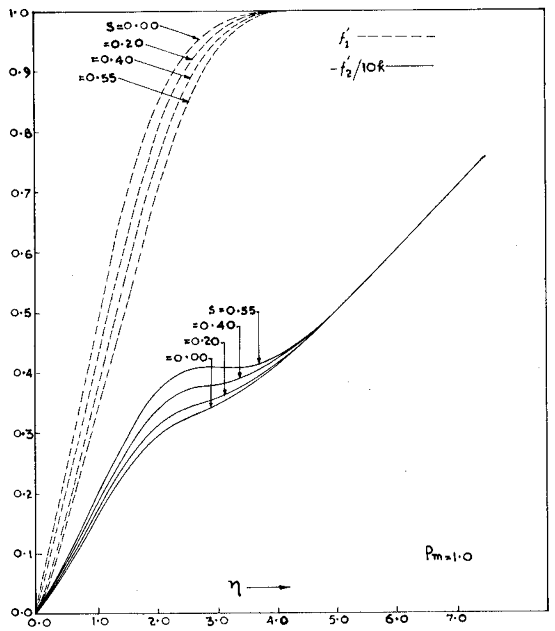

Figure 3. MHD flow over magnetised surface. Boundary layer velocity profiles.

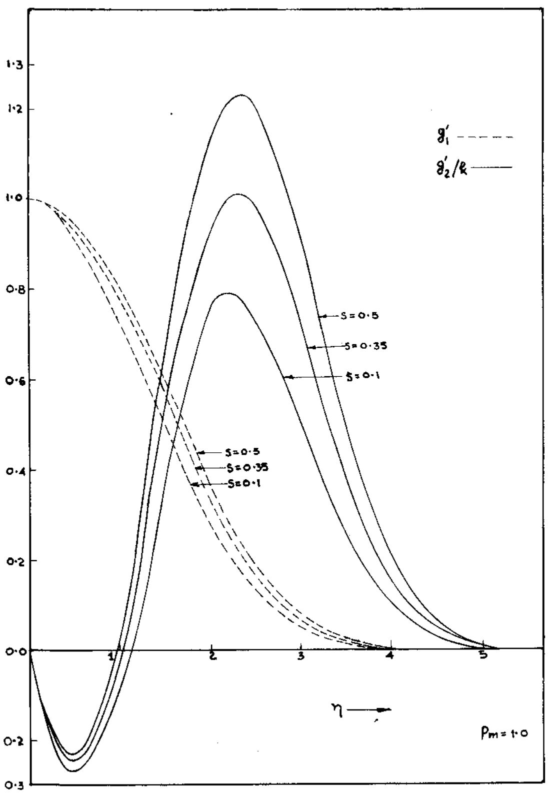

Figure 4. MHD flow over magnetised surface. Magnetic boundary layer profiles.

respectively, for different values of S and $Pm=1$. Table 2 gives the variation of the $O(1)$ and $O(\varepsilon)$ skin friction coefficients with S for different values of Pm.

The magnetic field reduces the $O(1)$ and $O(\varepsilon)$ skin friction coefficients on the convex side. The

TABLE 2

Variation in skin friction coefficient and initial values (case (ii))

Pm	S	$f_1''(0)$	$-g_1(0)$	$-f_2''(0)/k$	$-g_2(0)/k$
0.5	0.00	0.469600	1.958724	1.446967	1.296191
	0.10	0.427154	2.002760	1.451862	1.464298
	0.20	0.383292	2.053452	1.460700	1.681509
	0.30	0.337734	2.113079	1.476802	1.976301
	0.40	0.290054	2.185313	1.504132	2.400521
1.0	0.10	0.433787	1.570611	1.456612	1.249271
	0.20	0.396954	1.604218	1.471216	1.400750
	0.30	0.358845	1.642986	1.492128	1.598601
	0.40	0.319335	1.687924	1.518719	1.857241
	0.50	0.278064	1.741569	1.555615	2.217025
5.0	0.10	0.446405	0.905326	1.461310	0.814155
	0.20	0.422805	0.917800	1.476342	0.873779
	0.30	0.398569	0.931462	1.496795	0.945901
	0.40	0.373957	0.946230	1.517224	1.027584

relative effect of the magnetic field in reducing the $O(1)$ skin friction decreases with the increase in Pm. An opposite trend is observed in the case of $O(\varepsilon)$ skin friction. For large values of S, a moderate change in Pm appears to have very little influence on the $O(\varepsilon)$ skin friction.

An examination of the expression for the $O(\varepsilon)$ pressure suggests that the magnetic field should help the curvature in reducing the skin friction.

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